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High energy muons (⩾150 GeV) in extensive air showers

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Abstract. Results of an investigation on muons of energy greater than or equal to 150 GeV associated with extensive air showers are presented. The total number of muons is found to vary with shower size according to the relation

$$n_\mu(\geqslant 150, N) = (27\pm7)\left(\frac{N}{10^5}\right)^{0\cdot47\pm0\cdot05}$$

for $10^5 \leqslant N \leqslant 5\times10^6$. An increase in the power index $\alpha = \partial \ln n_\mu/\partial \ln N$ is indicated for $N < 10^5$. The energy spectrum of the muons is found to be a power law of the type $E_\mu^{-1\cdot30\pm0\cdot16}$ for $150\,\mathrm{GeV} \leqslant E_\mu \leqslant 640\,\mathrm{GeV}$. The observed variation of n_μ with N is found to be flatter than the one predicted by known EAS models for constant primary composition. A primary composition varying with primary energy and/or an energy dependent change in characteristics of nuclear interaction at high energies is suggested to account for the observed results.

1. Introduction

An investigation to study high energy muons (energy ⩾ 150 GeV) associated with extensive air showers has been carried out at Kolar Gold Fields, India, in collaboration with the Tata Institute of Fundamental Research (TIFR), Bombay. The basic aim of such an experiment is to derive information about the characteristics of high energy muons in EAS and thereby obtain information regarding the nuclear interactions and the composition of the primary cosmic rays at high energies. Various aspects of such a study have been discussed by Sreekantan (1963). The present paper deals with the variation of the total number of muons with shower size.

2. Experimental arrangement

The details of the experimental arrangement, utilized in the present investigation, have been given by Chowdhury and Saxena (1971). Basically the experimental set-up consists of: (i) an air shower array at the surface, consisting of 20 plastic scintillators arranged along the peripheries of concentric circles with increasing radii, and (ii) a penetrating particle detector.

The air shower array, at the surface, forms part of the experimental set-up of TIFR at Kolar Gold Fields (KGF) and has been described by Chatterjee *et al* (1965). The penetrating particle detector consists of a scintillator of area 1·5 m × 1·5 m and two trays of neon flash tubes. The detector is situated at a depth of 194 m underground corresponding to a depth of 580 hg cm^{-2} of KGF rock and the minimum energy required by the muon to penetrate this depth is about 150 GeV. The air shower array records showers within the size range 5×10^4–5×10^6 particles approximately.

Showers recorded with two different trigger requirements have been used for the present investigation.

2.1. S-trigger showers

These showers, which are identical with the S-trigger events described by Chatterjee *et al* (1968), were recorded with a selection criterion of a fourfold coincidence between any four surface scintillators when the recorded density exceeds a preset value in the inner ten detectors.

2.2. SU7-trigger showers

The selection criterion for these showers required a coincidence between the air shower pulse (ie the S-trigger pulse) and a pulse from the underground penetrating particle detector. These showers thus record at least one muon of energy greater than or equal to 150 GeV in association with an EAS.

3. Results

About 4000 showers have been recorded using the SU7-trigger. The rate of chance coincidence for these showers is estimated to be of the order of 1 per day. Taking into account the detection efficiency of the underground detector the total rate of SU7-triggers is found to be $(0{\cdot}92 \pm 0{\cdot}02)$ m^{-2} h^{-1}. For the present analysis use has also been made of about 10000 S-trigger showers.

3.1. The size spectra

Using the detection areas for showers of various sizes and taking the total effective running time into consideration the differential size spectra for S-trigger showers and SU7-trigger showers have been calculated. The spectra have been fitted to a power law and the power indices for the two spectra are given in table 1.

Table 1. Power indices of differential size spectra

Nature	S-trigger showers	SU7-trigger showers
Power index	$2{\cdot}78 \pm 0{\cdot}04$	$2{\cdot}30 \pm 0{\cdot}09$

The indices quoted in table 1 are for the size range $10^5 \leqslant N \leqslant 5 \times 10^6$. The SU7-trigger spectrum indicates a flattening in the slope for $N < 10^5$. The difference in the power indices γ and γ' of the S-trigger spectrum and SU7-trigger spectrum is given by

$$\alpha = \gamma - \gamma' = 0{\cdot}48 \pm 0{\cdot}02 \tag{1}$$

and can be accounted for by the increase in the probability of association of a shower, with a muon at underground level, with the shower size. A size dependence of muon number n_μ ($\geqslant 150, N$) of the following type is indicated:

$$n_\mu(\geqslant 150, N) \propto N^{0\cdot 48 \pm 0\cdot 02}. \tag{2}$$

However, a more exact relation between $n_\mu(\geqslant 150, N)$ and N can be obtained from the spectra given above using the procedure outlined by Chatterjee *et al* (1968) and given in following section.

3.2. *Absolute number of muons*

The rate of arrival of showers of size N in the interval $\mathrm{d}N$ within a solid angle Ω and having cores lying in the area A', can be written as

$$S(N)\,\mathrm{d}N = \int_\Omega \int_{A'} F(N)\,\mathrm{d}N \cos^n\theta\,\mathrm{d}\Omega\,\mathrm{d}A' \cos\theta. \tag{3}$$

Here $F(N)\,\mathrm{d}N$ represents the vertical flux of showers having size between N and $N+\mathrm{d}N$. The factor $\cos^n\theta$ arises because of the zenith angle dependence of the shower flux. Similarly the rate of arrival of showers, associated with a muon at the underground detector and having cores within an area A' at the surface and A at the underground plane, can be expressed as

$$SU(N)\,\mathrm{d}N = \int_A \int_{A'} F(N)\,\mathrm{d}N \cos^n\theta \frac{\mathrm{d}A \cos\theta\,\mathrm{d}A' \cos\theta}{l^2}\{1-\exp(-\Delta_\mu s \cos\theta)\} \tag{4}$$

where l is the distance between the elemental areas $\mathrm{d}A$ and $\mathrm{d}A'$ along the shower axis, s is the area of the underground detector and $\Delta_\mu(r)$ is the density of muons at the underground detector. $\Delta_\mu(r)$ essentially represents the density of muons of energy greater than or equal to E_μ in the air shower at a distance r from the shower axis, E_μ being the minimum energy required by muons to arrive at the underground detector. $\Delta_\mu(r)$ may be expressed as

$$\Delta_\mu(r) = \frac{n_\mu(\geqslant E_\mu, N)}{2\pi r_0^2} \exp\left(\frac{-r}{r_0}\right) \tag{5}$$

where $n_\mu(\geqslant E_\mu, N)$ is the total number of muons of energy greater than or equal to E_μ in a shower of size N and r_0 is a function of energy E_μ. The probability of association of a muon of energy greater than or equal to E_μ with EAS can be written as

$$P(N) = \frac{SU(N)\,\mathrm{d}N}{S(N)\,\mathrm{d}N}. \tag{6}$$

The right hand side of equation (6) was calculated numerically using equations (3)–(5) using $n_\mu(\geqslant E_\mu, N)$ and r_0 as free parameters. The variation of $P(N)$ with n_μ for six different values of r_0 is shown in figure 1. Values of $P(N)$ were also obtained for various shower sizes using the experimentally obtained size spectra. A comparison of the experimental probabilities with the ones given in figure 1 could yield the values of $n_\mu(\geqslant 150, N)$ provided the value of r_0 relevant to E_μ = 150 GeV is known.

An estimate of value of r_0 for E_μ = 150 GeV was made using the experimental values of mean lateral spreads for muons of different energies as obtained by Sivaprasad (1970)

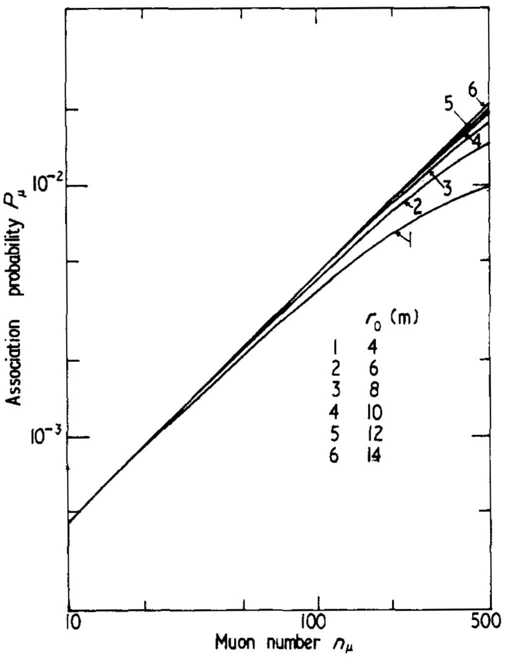

Figure 1. Variation of the association probability $P(N)$ with the number of muons n_μ.

and Barrett *et al* (1952) and as derived from the data of Earnshaw *et al* (1968). The value was found to be approximately 12 m for $E_\mu = 150$ GeV.

It is seen from figure 1 that the $P(N)$–n_μ relation is not very sensitive to the value of r_0 for the probability range $10^{-4} \leqslant P \leqslant 5 \times 10^{-3}$ (the range of the experimentally obtained values of $P(N)$) and for $8\text{ m} \leqslant r_0 \leqslant 14\text{ m}$. Thus a slight error in the r_0 value will not have any significant effect on the n_μ values.

Using the $P(N)$–n_μ curve for $r_0 = 12$ m, the values of $n_\mu(\geqslant 150, N)$ were obtained and are shown in figure 2.

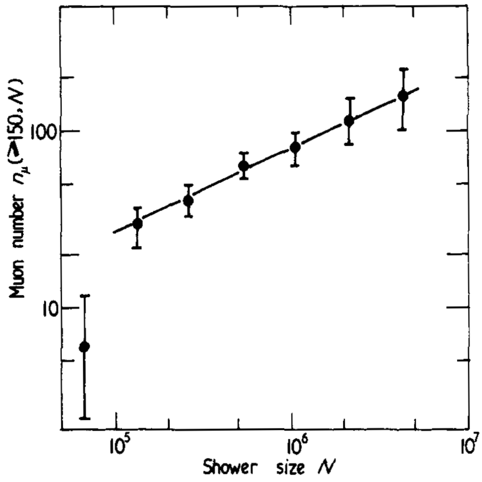

Figure 2. Variation of the total number of muons of energy greater than or equal to 150 GeV with the shower size.

The variation of $n_\mu(\geqslant 150, N)$ with N can be represented by

$$n_\mu(\geqslant 150, N) = (27 \pm 7)\left(\frac{N}{10^5}\right)^{0\cdot47 \pm 0\cdot05} \tag{7}$$

for $10^5 \leqslant N \leqslant 5 \times 10^6$. For $N < 10^5$ particles there is an indication of an increase in the slope $\alpha = \partial \ln n_\mu / \partial \ln N$. However the $n_\mu(\geqslant 150, N)$ value for $N < 10^5$ particles is subjected to large errors.

3.3. *Energy spectrum of muons*

The integral energy spectrum of muons of energy greater than or equal to 150 GeV has been obtained using the result of the present experiment along with the results of Sivaprasad (1970) for muons of energy greater than or equal to 220 GeV and greater than or equal to 640 GeV. The spectrum, given in figure 3, can be represented by a power law of the type

$$n_\mu(\geqslant E_\mu, 10^5) = (27 \pm 7)\left(\frac{E_\mu}{150}\right)^{-1\cdot30 \pm 0\cdot16} \tag{8}$$

where E_μ is expressed in GeV, and $E_\mu \geqslant 150$ GeV.

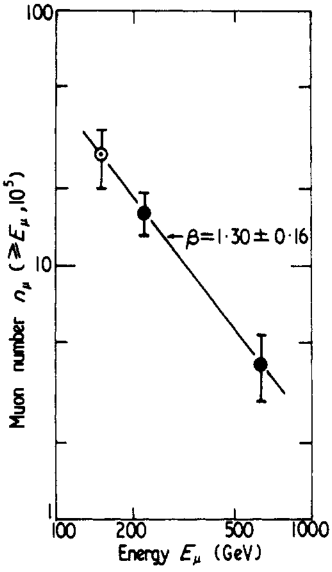

Figure 3. The integral energy spectrum of muons in EAS. ⊙ Present experiment; ● Sivaprasad (1970).

3.4. *Results of Monte Carlo calculations*

In order to compare the results presented above with the predictions of some of the models of EAS development Monte Carlo calculations based on these models were carried out. The models used were identical to the models, of identical names, used by Murthy *et al* (1968). The details are given in table 2. The results from the calculations may be summarized as follows.

Regarding the variation of the total number of muons (n_μ) with the shower size (N) for various threshold energies of the muons it is seen that all the models yield a power law relation with the power index α having values in the range 0·6–0·8. For muons of threshold energies less than or equal to 40 GeV QLN and IBN models predict muon numbers which are greater than the muon numbers predicted by QL and IB models respectively. However, at larger threshold energies the QLN and IBN models now yield numbers which are less than the corresponding numbers predicted by QL and IB models.

Figure 4 gives the variation of the number of muons of energy greater than or equal to 150 GeV with the shower size in the size range 10^5–5×10^6 particles. The variation is well represented by a power law with power index having the range 0·62–0·82. These predictions are for the proton initiated showers. Figure 5 gives the predicted variation of the number of muons of energy greater than or equal to 150 GeV for a mixed primary composition which remains constant throughout the primary energy range considered.

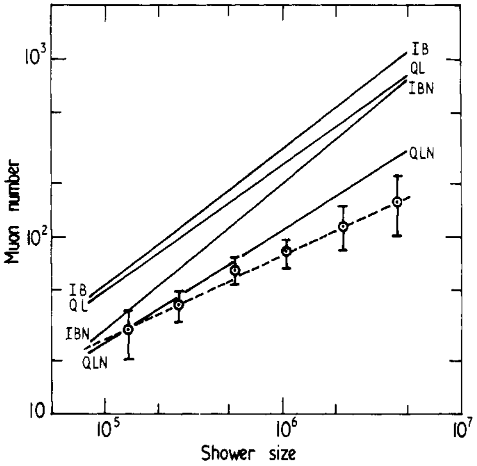

Figure 4. Predictions of the Monte Carlo calculations, on muons of energy greater than or equal to 150 GeV for proton primaries and experimental results (○).

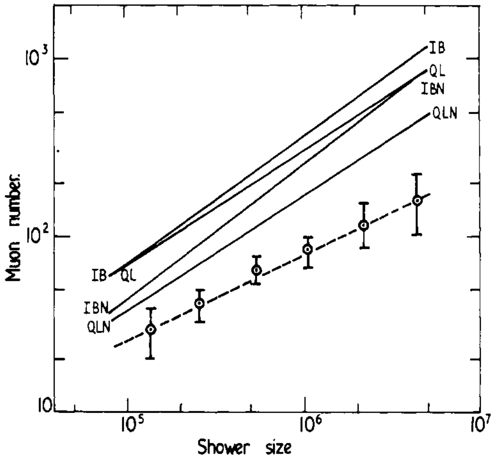

Figure 5. Predictions of the Monte Carlo calculations on muons of energy greater than or equal to 150 GeV for a mixed (constant) primary composition and experimental results (○).

The assumed composition is similar to the one estimated by the Balloon borne emulsion experiments at 10^{12} eV and is given in table 3. It is seen that the power index for the n_μ–N relationship in the case of the mixed composition lies in the range of 0·60–0·77 and the predicted number of muons for a given size and for a given model is larger than the corresponding number predicted by the model for proton primaries.

Table 2. Models of EAS development

	Model QLN		Model IBN		
	Nucleon	Pion	Nucleon		Pion
			Fireball	Isobar	
Multiplicity	$2{\cdot}7\,E^{1/4}$		$0{\cdot}25\,E^{1/2}$	3	$0{\cdot}96\,E^{1/2}$
Inelasticity	0·5	1·0	0·2	—	1
Mean free path ($\mathrm{g\,cm^{-2}}$)	80	120	80	—	120
Fraction of $N\bar{N}$ produced f	$\{7(500/E+1)\}^{-1}$		$\{7(500/E+1)\}^{-1}$		$\{7(500/E+1)\}^{-1}$
Energy spectrum of created particles	exponential		exponential	decided by kinematics	exponential

E is the energy of the projectile in GeV. Models QL and IB are identical to models QLN and IBN respectively except that for QL and IB models $f = 0$.

Table 3. Primary mass composition

Mass number of nucleus	Percentage composition
1	49
4	27
14	12
32	5
56	7

4. Discussion

4.1. Comparison of present experimental results with results of other experiments

Experimental results of Barrett *et al* (1952) and the TIFR EAS group (Sivaprasad 1970, Sreekantan 1971), may be compared directly with the results of the present experiment. Barrett *et al* (1952) studied muons of energy greater than or equal to 560 GeV in association with EAS and obtained the following dependence of the absolute number (n_μ) of muons on the shower size (N):

$$n_\mu(\geqslant 560, N) \propto N^{0\cdot45\pm0\cdot13}.$$

The experimental set-up used by Barrett *et al* did not yield information about the size of the recorded showers. Sivaprasad (1970) gave the following relations for muons of energy greater than or equal to 220 GeV and greater than or equal to 640 GeV in EAS in the size range $10^5 \leqslant N \leqslant 10^6$:

$$n_\mu(\geqslant 220, N) = (16 \pm 3)(N/10^5)^{0\cdot41\pm0\cdot09} \tag{10}$$

$$n_\mu(\geqslant 640, N) = (4\cdot1 \pm 1\cdot2)(N/10^5)^{0\cdot41\pm0\cdot15}. \tag{11}$$

There appears to be a reasonable agreement between the power indices obtained in the present experiment and in the experiments mentioned above. As far as the absolute number of muons is concerned, there is good agreement between the number of muons of energy greater than or equal to 150 GeV obtained on the basis of equations (10) and (11) and the results of the present experiment.

Greisen (1960) gave the following relation for the experimental results of Barrett *et al*:

$$n_\mu(\geqslant 560, N) = 75(N/10^6)^\alpha \tag{12}$$

where $\alpha \simeq 0\cdot7$ decreasing towards 0·5 for small values of n_μ and N. Considering the other available results Greisen (1960) also gave the energy spectrum for the muons associated with EAS. The spectrum turned out to be a power law with index $\beta \simeq -1\cdot37$. Though this index agrees well with the one obtained in the present experiment the number of muons obtained from equation (12) for $E_\mu \geqslant 560$ GeV turns out to be much higher than expected on the basis of present results.

The discrepancy appears to arise from the fact that the absolute number of muons in equation (12) is obtained by matching the absolute flux of the muons at the observation level to the air shower flux, and assigning one muon of energy greater than or equal to 560 GeV to showers of size N for which the integral flux of the air showers is the same as the absolute flux of the muons at the observation level. On this basis Barrett *et al* (1952) concluded that there is one muon of energy greater than or equal to 560 GeV in a shower of size about 400 electrons. There is a flaw in this argument of transition from matching the absolute muon flux with the air shower flux to assigning one muon of energy greater than or equal to 560 GeV to showers of sizes about 400 electrons, as has been pointed out by Sivaprasad (1970). Assuming that every primary particle of energy E_p (corresponding to the shower size N_0) is associated with one muon of energy greater than or equal to 560 GeV, the steady state flux of these muons can be obtained by integrating over all primary energies. Because of a steep primary energy spectrum and an E_p^α ($\alpha < 1$) type of dependence of muon number on E_p, a dominant contribution to the muon flux will come from the particles of primary energies less than or equal to E_p. Therefore, the procedure of assigning one muon of energy greater than or equal to 560 GeV to shower size N, for which the integral shower flux and the total flux of muons observed at underground level match is not correct.

Asekin *et al* (1971) have obtained the following muon spectrum, for muons associated with EAS:

$$n_\mu(\geqslant E_\mu, N) = (3\cdot1 \pm 1\cdot5)(E_\mu/0\cdot3)^{-\beta}(N/3\times10^4)^\alpha \tag{13}$$

where

$$\beta = 1\cdot58 \pm 0\cdot20$$

$$\alpha = 0\cdot68 + 0\cdot24$$

for $N \geqslant 3\times10^4$ and $E_\mu \geqslant 0\cdot3$ TeV. The values of the exponents of both energy and size spectra are larger than obtained in the present experiment, but the present results are within the error limits given by Asekin *et al* (1971).

The results for muons of low threshold energies ($\leqslant 40$ GeV) indicate a steeper variation ($\alpha \simeq 0{\cdot}75$) than the one obtained in the experiments discussed above (Greisen 1960, Nikol'skii 1962, Earnshaw *et al* 1968, Khrenov 1965, Hara *et al* 1970, Vernov *et al* 1970).

4.2. *Comparison of the experimental results with the predictions of models*

The results from the present experiment are shown along with the predictions of the Monte Carlo calculations in figures 4 and 5. Whereas the experiment gives a power index of ($0{\cdot}47 \pm 0{\cdot}05$) for the n_μ–N variation for muons of energy greater than or equal to 150 GeV, the models predict power indices which are much larger than the experimental value. The absolute number of muons is closer to the predicted value for showers of 10^5 particles and the discrepancy increases with the increasing shower size because of the difference in power indices. Thus we see that the models under consideration cannot reproduce the experimentally observed variation of n_μ with N for protons as well as the mixed composition primaries.

Two possible explanations could be envisaged for the observed n_μ–N variation being flatter than the one predicted by the models. The first explanation is to invoke an energy dependent change in some characteristic of the interaction, in the relevant energy region, which may lead to a decrease in the fraction of energy going into the production of the parent particles of muons. This will lead to a relative reduction in the muon number for higher shower sizes and will result in a flatter n_μ–N variation. An alternative explanation is to invoke a change in the mass composition of the primary cosmic rays in the relevant energy region such that the average mass number $\langle A \rangle$ of the primaries decreases from a high value for small showers to lower values at larger shower sizes.

4.3. *Change in the nature of the characteristics of the nuclear interactions and the n_μ–N relation*

One of the possible changes which may be envisaged in the characteristics of the nuclear interactions leading to a flatter n_μ–N variation, is an energy dependent increase in the nucleon–antinucleon ($\mathrm{N\overline{N}}$) production. As seen earlier the production of $\mathrm{N\overline{N}}$ results in the reduction in muon number for muons of threshold energies beyond 40 GeV. Thus in the energy range relevant to the sizes 10^5–10^6 at the observation level, an energy dependent increase in the production of $\mathrm{N\overline{N}}$ may result in a flatter n_μ–N variation than predicted by the usual models.

There is no experimental evidence against the increased $\mathrm{N\overline{N}}$ production at high energies. Tonwar *et al* (1971) have shown that the $\mathrm{N\overline{N}}$ production at 10^{12} eV is as high as 14%. However, to fit the observational data above the size range 10^6, this effect of increase in $\mathrm{N\overline{N}}$ production with increasing primary energies must saturate at an energy region in the neighbourhood of 10^{16} eV.

4.4. *Primary mass composition and the n_μ–N relation*

Balloon borne experiments, using large nuclear emulsion stacks, indicate that the composition of the primary cosmic rays remains unaltered up to about 10^{12} eV. At very high energies ($\simeq 10^{17}$ eV), the smallness of the fluctuations in the characteristics of the EAS indicates a pure protonic nature of the primaries as discussed by Linsley and Scarci (1962) and Suga *et al* (1970). A change in the primary chemical composition may then be expected in the intervening energy interval (10^{12}–10^{16}) eV. It has long been considered that the primary cosmic rays over a certain threshold of the magnetic rigidity may not be retained in our galaxy and there may exist a galactic rigidity cut-off for the primary cosmic rays, beyond which the cosmic rays may be of extragalactic origin.

Cowsik (1968) calculated the size dependence of the average shower characteristics, using an 'isobar-cum-fireball' model, similar to the model of Pal and Peters (1964), and assuming a changing composition due to galactic rigidity cut-off. He assumed a rigidity cut-off at 10^5 GV and a chemical composition the same as at low energies up to the cut-off. Beyond the cut-off a pure extragalactic proton primary component was assumed. A comparison of the results from the present experiment with the results of Cowsik (1968) is given in figure 6. Besides the disagreement with the absolute values, the observed trend of the variation is not reproduced under the assumptions and the model used by Cowsik (1968). Chatterjee (1964) also proposed a similar model for the primary energy spectrum using galactic cut-off for protons at 3×10^{14} eV and Z times higher for the heavier particles. Sivaprasad (1970) has modified this model slightly by introducing a gradual cut-off starting at 10^{14} eV for the protons. The flux was assumed to drop off to 1% of the pre-cut-off value within an energy equal to 2·5 times the cut-off energy. Using this type of primary spectrum and the models identical to those used by Murthy *et al* (1967), Sivaprasad (1970) studied the n_μ–N variation and found that the predicted trend of this variation is in reasonable agreement with his experimental results for muons of energy greater than or equal to 220 GeV and greater than or equal to 640 GeV. The predictions are not in conflct with the experimental results for muons of threshold energies up to 40 GeV.

Results of the calculations by Sivaprasad (1970) for muons of energy greater than or equal to 220 GeV are shown in figure 7 along with the results of the present experiment for muons of energy greater than or equal to 150 GeV. The trend of the n_μ–N variation, predicted by the models is not in disagreement with the observed trend. However, the predicted numbers are larger than the observed number of muons.

The effect of the magnetic rigidity cut-off on the n_μ–N relation can be understood as follows. For a given level of observation, the average size of the showers initiated by a heavy primary will be different from the size of the shower initiated by a proton of the

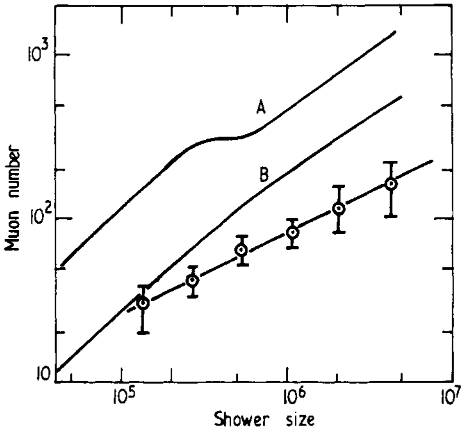

Figure 6. Comparison of the experimental results with the calculations of Cowsik (1968) for a mixed primary composition subjected to a sharp rigidity cut-off. Curves A and B are from Cowsik (1968) with $E_\mu \geqslant 100$ GeV and 200 GeV respectively. ⊙ Present experiment, $E_\mu \geqslant 150$ GeV.

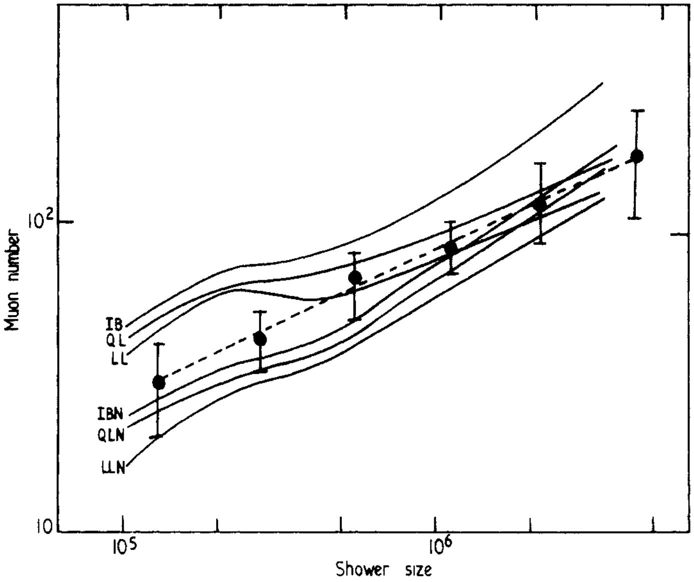

Figure 7. Comparison of the experimental results with the calculations of Sivaprasad (1970) for a mixed primary composition subjected to a gradual rigidity cut-off. Full curves are from the calculations of Sivaprasad for $E_\mu \geqslant 220$ GeV. The full circles with the broken curve are from the present experiment with $E_\mu \geqslant 150$ GeV.

same energy. Thus the primary energy required to produce a shower of a given size, will be different for the primaries of different 'A' values. It is this difference in the primary energy, for producing showers of a given size, coupled with the existence of the rigidity cut-off that leads to a decrease in $\langle A \rangle$ with the increase in the shower size, even though there is an increase in $\langle A \rangle$ with increasing primary energy. This in turn leads to a rather flat n_μ–N variation than that obtained on the basis of a constant composition. The Durham group (de Beer *et al* 1968, Adcock *et al* 1968) has carried out a theoretical study of the possible consequences, of a primary composition subjected to a magnetic rigidity cut-off, on various parameters of EAS. For such a composition the authors anticipate a highly characteristic oscillation in the value of $\alpha = \partial \ln n_\mu / \partial \ln N$ as a function of N. Catz *et al* (1970) have observed a somewhat similar oscillation in the experimentally obtained values of α in the size range predicted by the Durham group. Though there are significant errors on the values of α as measured by Catz *et al* and though the fluctuations are not as much as anticipated by the Durham group, the results of Catz *et al* appear to support a changing primary composition subjected to magnetic rigidity cut-off. However, Thompson *et al* (1970), from an examination of recent data on muons in EAS, conclude that the majority of the primaries are still protons above 10^{15} eV.

5. Conclusions

From the above considerations it can be concluded that the n_μ–N variation as observed in the present experiment for high energy muons ($\geqslant 150$ GeV) is flatter than the one predicted by the models known at present using a constant primary composition.

It is seen that a primary composition subjected to a gradual galactic rigidity cut-off may reproduce the observed variation. At this point it may be pointed out that if the observed n_μ–N variation in this size range is due to a rigidity cut-off in the relevant primary energy range then outside this range the n_μ–N variation for the muons of higher energies should be a power law with index $\alpha \simeq 0{\cdot}6$–$0{\cdot}8$. This is expected because outside the cut-off region a constant primary composition exists and such a composition will result in a power law index $\alpha \simeq 0{\cdot}6$–$0{\cdot}8$. It is, therefore, necessary to study the variation of muon number with shower size over a wide shower size range for muons of various threshold energies.

The possibility of a change in the characteristic of the nuclear interaction with the energy, however, cannot be ruled out.

Acknowledgments

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